KINDERGARTEN MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

K MATH

v.4.04

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

DATA & PROBABILITY

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KINDERGARTEN

The big ideas for kindergarten children are in the area of number. The Expectations at Kindergarten emphasize counting, grouping, and ordering numbers. Teachers should emphasize meaning, naming, and patterns.

NUMBER AND OPERATIONS

Count, write, and order numbers

N.ME.00.01 Count whole numbers and recognize how many objects are in sets to 30.

N.ME.00.02 Use one-to-one correspondence to compare and order sets of objects to 30 using such phrases as "same number", "more than", or "less than"; use counting and matching.

N.ME.00.03 Compare and order numbers to 30 using phrases such as "more than" or "less than."

N.ME.00.04 Read and write numerals to 30 and connect them to the quantities they represent.

N.ME.00.05 Count orally to 100 by ones. Count to 30 by 2s, 5s and10s using grouped objects as needed.

Compose and decompose numbers

N.ME.00.06 Understand the numbers 1 to 30 as having one, or two, or three groups of ten and some ones. Also count by tens with objects in ten-groups to 100.

N.MR.00.07 Compose and decompose numbers from 2 to 10, e.g., 5= 4 +1 = 2+3, with attention to the additive structure of numbers, e.g., 6 is 1 more than 5, 7 is one more than 6.

N.MR.00.08 Describe and make drawings to represent situations/stories involving putting together and taking apart for totals up to 10; use finger and object counting.

Add and subtract numbers

N.MR.00.09 Record mathematical thinking by writing simple addition and subtraction sentences, e.g., 7 + 2 = 9, 10 - 8 = 2.

Explore number patterns

N.MR.00.10 Create, describe, and extend simple number patterns.

MEASUREMENT

Explore concepts of time

M.UN.00.01 Know and use the common words for the parts of the day (morning, afternoon, evening, night) and relative time (yesterday, today, tomorrow, last week, next year).

M.TE.00.02 Identify tools that measure time (clocks measure hours and minutes; calendars measure days, weeks, and months).

M.UN.00.03 Identify daily landmark times to the nearest hour (lunchtime is 12 o'clock; bedtime is 8 o'clock)

Explore other measurement attributes

M.UN.00.04 Compare two or more objects by length, weight and capacity, e.g., which is shorter, longer, taller?

M.PS.00.05 Compare length and weight of objects by comparing to reference objects, and use terms such as shorter, longer, taller, lighter, heavier.

GEOMETERY

Create, explore, and describe shapes

G.GS.00.01 Relate familiar three-dimensional objects inside and outside the classroom to their geometric name, e.g., ball/sphere, box/cube, soup can/cylinder, ice cream cone/cone, refrigerator/prism.

G.GS.00.02 Identify, sort and classify objects by attribute and identify objects that do not belong in a particular group.

Explore geometric patterns

G.GS.00.03 Create, describe, and extend simple geometric patterns.

FIRST GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

1 MATH

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FIRST GRADE

In the First Grade Expectations, children are asked to consolidate their counting knowledge, and to explore addition and subtraction as number operations. Work in Number and Operations will draw heavily on the use of concrete materials and contextual situations that make sense to children. Students should be fluent with addition and subtraction facts up to 10+10 by the end of first grade. Measurement is introduced through time and money, and geometric ideas are based on children's experiences in the real world.

NUMBER AND OPERATIONS

Count, write, and order numbers

N.ME.01.01 Count to 110 by 1's, 2's, 5's, and 10's, starting from any number in the sequence; count to 500 by 100s and 10s; use ordinals to identify position in a sequence, e.g., 1st, 2nd, 3rd.

N.ME.01.02 Read and write numbers to 110 and relate them to the quantities they represent.

N.ME.01.03 Order numbers to 110; compare using the phrases: same as, more than, greater than, fewer than; use = symbol. Arrange small sets of numbers in increasing or decreasing order, e.g., write the following from smallest to largest: 21, 16, 35, 8.

N.ME.01.04 Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.

N.ME.01.05 Understand that a number to the right of another number on the number line is bigger and that a number to the left is smaller.

N.ME.01.06 Count backward by 1's starting from any number between 1 and 100.

Explore place value

N.ME.01.07 Compose and decompose numbers to 30 including using bundles of tens and units, e.g., recognize 24 as 2 tens and 4 ones, 10 and 10 and 4, 20 and 4, and 24 ones.

Add and subtract whole numbers

N.ME.01.08 List number facts (partners inside of numbers) for 2 through 10; e.g., 8 = 7 + 1 = 6 + 2 = 5 + 3 = 4 + 4; 10 = 8 + 2 = 2 + 8.

N.MR.01.09 Compare two or more sets in terms of the difference in number of elements.

N.MR.01.10 Model addition and subtraction for numbers less than 20 for a given contextual situation using objects or pictures; explain in words; record using numbers and symbols; solve.

N.MR.01.11 Understand the inverse relationship between addition and subtraction, e.g., subtraction "undoes" addition: if 3 + 5 = 8, we know that 8 - 3 = 5 and 8 - 5 = 3; recognize that some problems involving combining, "taking away," or comparing can be solved by either operation.

N.FL.01.12 Know all the addition facts up to 10 + 10, and solve the related subtraction problems fluently.

N.MR.01.13 Apply knowledge of fact families to solve simple open sentences for addition and subtraction, such as: ☐ + 2 = 7 and 10 - ☐ = 6.

N.FL.01.14 Add three one-digit numbers.

N.FL.01.15 Calculate mentally sums and differences involving: a two-digit number and a one-digit number without regrouping; a two-digit number and a multiple of 10.

N.FL.01.16 Compute sums and differences up to two-digit numbers using number facts and strategies, but no formal algorithm.

MEASUREMENT

Estimate and measure length

M.UN.01.01 Measure the lengths of objects in non-standard units, (e.g., pencil lengths, shoe lengths) to the nearest whole unit.

M.UN.01.02 Compare measured lengths using the words shorter, shortest, longer, longest, taller, tallest, etc.

MEASUREMENT

Tell time

M.UN.01.03 Tell time on a twelve-hour clock face to the hour and half-hour.

Work with money

M.UN.01.04 Identify the different denominations of coins and bills.

M.UN.01.05 Match one coin or bill of one denomination to an equivalent set of coins/bills of other denominations, e.g., 1 quarter = 2 dimes and 1 nickel.

M.PS.01.06 Tell the amount of money: in cents up to $1, in dollars up to $100. Use the symbols $ and ¢.

M.UN.01.07 Add and subtract money in dollars only or in cents only.

Solve problems

M.PS.01.08 Solve one-step word problems using addition and subtraction of length, money and time, including "how much more/less", without mixing units.

GEOMETRY

Create and describe shapes

G.GS.01.01 Create common two-dimensional and three-dimensional shapes, and describe their physical and geometric attributes, such as color and shape.

G.LO.01.02 Describe relative position of objects on a plane and in space, using words such as above, below, behind, in front of.

Create and describe patterns involving geometric objects

G.SR.01.03 Create and describe patterns, such as repeating patterns, and growing patterns using number, shape, and size.

G.SR.01.04 Distinguish between repeating and growing patterns.

G.SR.01.05 Predict the next element in a simple repeating pattern.

G.SR.01.06 Describe ways to get to the next element in simple repeating patterns.

DATA AND PROBABILITY

Use pictographs

D.RE.01.01 Collect and organize data to use in pictographs.

D.RE.01.02 Read and interpret pictographs.

D.RE.01.03 Make pictographs of given data using both horizontal and vertical forms of graphs; scale should be in units of one and include symbolic representations, e.g., ☺ represents one child.

SECOND GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

2 MATH

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SECOND GRADE

Second graders continue with more sophisticated work in addition and subtraction of whole numbers, and begin conceptual development of multiplication and division, grounded in contexts and modeled using concrete objects. Teachers can emphasize the inverse relationships between addition and subtraction, and multiplication and division. Children will be using strategies and algorithms, to compute using whole numbers. Simple ideas about fractions are introduced. In geometry, children continue to learn about geometric shapes and their elements.

NUMBER AND OPERATIONS

Count, write, and order whole numbers

N.ME.02.01 Count to 1000 by 1's, 10's and 100's starting from any number in the sequence.

N.ME.02.02 Read and write numbers to 1000 in numerals and words, and relate them to the quantities they represent.

N.ME.02.03 Compare and order numbers to 1000; use the symbols > and <.

N.ME.02.04 Count orally by 3's and 4's starting with 0, and by 2's, 5's and 10's starting from any number.

Understand place value

N.ME.02.05 Express numbers up to 1000 using place value, e.g., 137 is 1 hundred, 3 tens, and 7 ones; use concrete materials.

Add and subtract whole numbers

N.FL.02.06 Decompose 100 into addition pairs, e.g., 100 = 99 + 1 = 98 + 2…

N.MR.02.07 Find the distance between numbers on the number line, e.g., how far is 79 from 26?

N.MR.02.08 Find missing values in open sentences, e.g., 42 + □ = 57; use relationship between addition and subtraction.

N.MR.02.09 Given a contextual situation that involves addition and subtraction for numbers up to two digits,: model using objects or pictures, explain in words, record using numbers and symbols; solve.

N.FL.02.10 Add fluently two numbers up to two digits each, using strategies including formal algorithms; subtract fluently two numbers up to two digits each.

N.FL.02.11 Estimate and calculate the sum of two numbers with three digits that do not require regrouping.

N.FL.02.12 Calculate mentally sums and differences involving: three-digit numbers and ones; three-digit numbers and tens; three-digit numbers and hundreds.

Understand meaning of multiplication and division

N.MR.02.13 Understand multiplication as the result of counting the total number of objects in a set of equal groups, e.g., 3 x 5 gives the number of objects in 3 groups of 5 objects, or 3 x 5 = 5 + 5 + 5 = 15.

N.MR.02.14 Represent multiplication using area and array models.

N.MR.02.15 Understand division (÷) as another way of expressing multiplication, using fact families within the 5 x 5 multiplication table; emphasize that division "undoes" multiplication, e.g., 2 x 3 = 6 can be rewritten as 6 ÷ 2 = 3 or 6 ÷ 3 = 2.

N.MR.02.16 Given a simple situation involving groups of equal size or of sharing equally, represent with objects, words, and symbols; solve.

N.FL.02.17 Develop strategies for fluently multiplying numbers up to 5 x 5.

Work with unit fractions

N.ME.02.18 Recognize, name, and represent commonly used unit fractions with denominators 12 or less; model 1/2, 1/3, and 1/4 by folding strips.

N.ME.02.19 Recognize, name, and write commonly used fractions: 1/2, 1/3, 2/3, 1/4, 2/4, 3/4.

N.ME.02.20 Place 0 and halves, e.g., 1/2, $1^1/_2$, $2^1/_2$, on the number line; relate to a ruler.

N.ME.02.21 For unit fractions from 1/12 to 1/2, understand the inverse relationship between the size of the denominator; compare unit fractions from 1/12 to 1/2.

N.ME.02.22 Recognize that fractions such as 2/2, 3/3 and 4/4 are equal to the whole (one).

MEASUREMENT

Measure, add, and subtract length

M.UN.02.01 Measure lengths in meters, centimeters, inches, feet, and yards approximating to the nearest whole unit using abbreviations: cm, m, in, ft, yd.

G.GS.02.03 Draw rectangles and triangles, and compute perimeters by adding lengths of sides, recognizing the meaning of perimeter.

M.PS.02.02 Compare lengths; add and subtract lengths (no conversion of units).

Understand the concept of area

M.UN.02.03 Measure area using non-standard units to the nearest whole unit.

M.TE.02.04 Find the area of a rectangle with whole number side lengths by covering with unit squares and counting, or by using a grid of unit squares; write the area as a product.

Tell time and solve time problems

M.UN.02.05 Using both A.M. and P.M., tell and write time from the clock face in 5 minute intervals and from digital clocks to the minute; include reading time: 9:15 as nine-fifteen and 9:50 as nine-fifty. Interpret time both as minutes after the hour and minutes before the next hour, e.g., 8:50 as eight-fifty and ten to nine. Show times by drawing hands on clock face.

M.UN.02.06 Use the concept of duration of time, e.g., determine what time it will be half an hour from 10:15.

Record, add and subtract money

M.UN.02.07 Read and write amounts of money using decimal notations, e.g., $1.15.

M.PS.02.08 Add and subtract money in mixed units, e.g., $2.50 + 60 cents and $5.75 - $3, but not $2.50 + $3.10.

Read thermometers

M.UN.02.09 Read temperature using the scale on a thermometer in degrees Fahrenheit.

Solve measurement problems

M.PS.02.10 Solve simple word problems involving length and money.

GEOMETRY

Identify and describe shapes

G.GS.02.01 Identify, describe, and compare familiar two-dimensional and three-dimensional shapes such as triangles, rectangles, squares, circles, semi-circles, spheres and rectangular prisms.

G.GS.02.02 Explore and predict the results of putting together and taking apart two-dimensional and three-dimensional shapes.

G.GS.02.03 Draw rectangles and triangles, and compute perimeters by adding lengths of sides, recognizing the meaning of perimeter.

G.GS.02.04 Distinguish between curves and straight lines and between curved surfaces and flat surfaces.

G.SR.02.05 Classify familiar plane and solid objects, e.g., square, rectangle, rhombus, cube, pyramid, prism, cone, cylinder, and sphere, by common attributes such as shape, size, color, roundness, or number of corners and explain which attributes are being used for classification.

G.TR.02.06 Recognize that shapes that have been slid, turned or flipped are the same shape, e.g., a square rotated 45° is still a square.

Use coordinate systems

G.LO.02.07 Find and name locations using simple coordinate systems such as maps and first quandrant grids.

DATA AND PROBABILITY

Create, interpret, and solve problems involving pictographs

D.RE.02.01 Make pictographs using a scale representation, using scales where symbols equal more than one.

D.RE.02.02 Read and interpret pictographs with scales, using scale factors of 2 and 3.

D.RE.02.03 Solve problems using information in pictographs; include scales such as each ■ represents 2 apples; avoid ⬓ cases.

THIRD GRADE MATHEMATICS

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THIRD GRADE

In the third grade, students gain proficiency in addition and subtraction of whole numbers, and continue to develop meaning and computational skill in multiplication. This culminates in knowledge of the 10x10 multiplication table. Students are introduced to decimals through money. Work in measurement is closely related to increased emphasis on ideas from geometry including developing meaning for area and perimeter.

NUMBER AND OPERATIONS

Understand and use number notation and place value

N.ME.03.01 Read and write numbers to 10,000 in both numerals and words, and relate them to the quantities they represent, e.g., relate numeral or written word to a display of dots or objects.

N.ME.03.02 Recognize and use expanded notation for numbers using place value to 10,000s place, e.g., 2,517 is 2 thousands, 5 hundreds, 1 ten, and 7 ones; 4 hundreds and 2 ones is 402. Identify the place value of a digit in a number, e.g., in 3,241, 2 is in the hundreds place.

N.ME.03.03 Compare and order numbers up to 10,000.

Count in steps, and understand even and odd numbers

N.ME.03.04 Count orally by 6's, 7's, 8's and 9's starting with 0, making the connection between repeated addition and multiplication.

N.ME.03.05 Know that even numbers end in 0, 2, 4, 6,or 8; name a whole number quantity that can be shared in two equal groups or grouped into pairs with no remainders; recognize even numbers as multiples of 2. Know that odd numbers end in 1, 3, 5, 7, or 9, and work with patterns involving even and odd numbers.

Add and subtract whole numbers

N.FL.03.06 Add and subtract fluently two numbers: up to and including two-digit numbers with regrouping and up to four-digit numbers without regrouping.

N.FL.03.07 Estimate the sum and difference of two numbers with three digits (sums up to 1,000), and judge reasonableness of estimates.

N.FL.03.08 Use mental strategies to fluently add and subtract two-digit numbers.

Multiply and divide whole numbers

N.MR.03.09 Use multiplication and division fact families to understand the inverse relationship of these two operations, e.g., because $3 \times 8 = 24$, we know that $24 \div 8 = 3$ or $24 \div 3 = 8$. Express a multiplication statement as an equivalent division statement.

N.MR.03.10 Recognize situations that can be solved using multiplication and division including finding "How many groups?" and "How many in a group?" and write mathematical statements for those situations.

N.FL.03.11 Find products fluently up to 10×10; find related quotients using multiplication and division relationships.

N.MR.03.12 Find solutions to open sentences such as $7 \times \square = 42$ or $12 \div \square = 4$, using the inverse relationship between multiplication and division.

N.FL.03.13 Mentally calculate simple products and quotients up to a three-digit number by a one-digit number involving multiples of 10, e.g., 500×6, or $400 \div 8$.

N.MR.03.14 Solve simple division problems involving remainders, viewing remainder as the "number left over" (less than the divisor), e.g., 4 children per group; we have 25 children; there are 6 groups with 1 child left over; interpret based on problem context.

Problem-solving with whole numbers

N.MR.03.15 Given problems that use any one of the four operations with appropriate numbers, represent with objects, words (including "product" and "quotient"), and mathematical statements; solve.

Understand simple fractions, relation to the whole, and addition and subtraction of fractions

N.ME.03.16 Understand that fractions may represent a portion of a whole unit that has been partitioned into parts of equal area or length; use the terms "numerator" and "denominator."

N.ME.03.17 Recognize, name and use equivalent fractions with denominators 2, 4, and 8, using strips as area models.

N.ME.03.18 Place fractions with denominators of 2, 4, and 8 on the number line; relate the number line to a ruler; compare and order up to three fractions with denominators 2, 4, and 8.

N.ME.03.19 Understand that any fraction can be written as a sum of unit fractions, e.g., 3/4 = 1/4 + 1/4 + 1/4.

N.MR.03.20 Recognize that addition and subtraction of fractions with equal denominators can be modeled by joining and taking away segments on the number line.

Understand simple decimal fractions in relation to money

N.ME.03.21 Understand the meaning of $0.50 and $0.25 related to money, e.g., $1.00 shared by two people means $1.00 ÷ 2 = 1/2 dollar = $0.50.

MEASUREMENT

Measure and use units for length, weight, temperature and time

M.UN.03.01 Know and use common units of measurements in length, weight and time.

M.UN.03.02 Measure in mixed units within the same measurement system for length, weight and time: feet and inches, meters and centimeters, kilograms and grams, pounds and ounces, liters and milliliters, hours and minutes, minutes and seconds, years and months.

M.UN.03.03 Understand relationships between sizes of standard units, e.g., feet and inches, meters and centimeters.

M.UN.03.04 Know benchmark temperatures such as freezing (32°F, 0°C); boiling, (212°F, 100°C); and compare temperatures to these, e.g., cooler, warmer.

Understand meaning of area and perimeter and apply in problems

M.UN.03.05 Know the definition of area and perimeter and calculate the perimeter of a square and rectangle given whole number side lengths.

M.UN.03.06 Use square units in calculating area by covering the region and counting the number of square units.

M.UN.03.07 Distinguish between units of length and area and choose a unit appropriate in the context.

M.UN.03.08 Visualize and describe the relative sizes of one square inch and one square centimeter.

Estimate perimeter and area

M.TE.03.09 Estimate the perimeter of a square and rectangle in inches and centimeters; estimate the area of a square and rectangle and square inches and square centimeters.

Solve measurement problems

M.PS.03.10 Add and subtract lengths, weights and times using mixed units within the same measurement system.

M.PS.03.11 Add and subtract money in dollars and cents.

M.PS.03.12 Solve applied problems involving money, length and time.

M.PS.03.13 Solve contextual problems about perimeters of rectangles and areas of rectangular regions.

GEOMETRY

Recognize the basic elements of geometric objects

G.GS.03.01 Identify points, line segments, lines and distance.

G.GS.03.02 Identify perpendicular lines and parallel lines in familiar shapes and in the classroom.

G.GS.03.03 Identify parallel faces of rectangular prisms, in familiar shapes and in the classroom.

Name and explore properties of shapes

G.GS.03.04 Identify, describe, compare and classify two-dimensional shapes, e.g., parallelogram, trapezoid, circle, rectangle, square and rhombus, based on their component parts (angles, sides, vertices, line segment) and the number of sides and vertices.

G.SR.03.05 Compose and decompose triangles and rectangles to form other familiar two-dimensional shapes, e.g., form a rectangle using two congruent right triangles, or decompose a parallelogram into a rectangle and two right triangles.

Explore and name three-dimensional solids

G.GS.03.06 Identify, describe, build and classify familiar three-dimensional solids, e.g., cube, rectangular prism, sphere, pyramid, cone, based on their component parts (faces, surfaces, bases, edges, vertices).

G.SR.03.07 Represent front, top, and side views of solids built with cubes.

DATA AND PROBABILITY

Use bar graphs

D.RE.03.01 Read and interpret bar graphs in both horizontal and vertical forms.

D.RE.03.02 Read scales on the axes and identify the maximum, minimum, and range of values in a bar graph.

D.RE.03.03 Solve problems using information in bar graphs including comparison of bar graphs.

FOURTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

4 MATH

v.4.04

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

DATA & PROBABILITY

Welcome to a preview of Michigan's mathematical future! This document not only introduces Michigan's new Grade Level Content Expectations for mathematics, it also establishes high expectations in mathematics to better prepare all K-12 Michigan students for the challenges of the future.

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This document is intended to be an assessment tool. This means students will be expected to be proficient in the concepts and skills included in this document at the end of the indicated grade level. These expectations are written to convey intended performances by students. The expectations here generally represent key landmarks in mathematics learning — areas where students are expected to have consolidated their understandings and skills. Thus it does not attempt to elaborate all of the precursor ideas and concepts that lead to a particular expectation in a particular grade level — it instead assumes that teachers will build up to the expectations through exploration and development of concepts and processes

The Grade Level Content Expectations are not designed to be a curriculum document, or to function as a scope and sequence framework. It is not designed to suggest the various pedagogical options and strategies that might best enable students to attain these expectations. Rather, it should serve as a basis for the development of a curriculum and instructional strategies that would help the students attain the concepts and skills necessary to meet the GLCE. Various groups are being organized

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to develop clarification documents, content examples, more elaborated explanations, and suggestions for professional development that would support these expectations. Ultimately, teachers, school personnel and district leaders will need to collaborate and draw on their own professional wisdom and experience, as well as on research, to decide how best to organize instruction to help their students meet these expectations.

The mathematics content expectations have been organized into five strands: Number and Operations, Algebra, Geometry, Measurement, and Data and Probability.

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FOURTH GRADE

By the end of fourth grade students will have consolidated addition and subtraction of whole numbers, and will have done much in multiplication and division of whole numbers. Work in number also extends to fractions and decimal fractions, using limited sets of fractions as the basis for building meaning for equivalent fractions, addition, subtraction, and fraction as part of a set of objects. Work in measurement becomes more sophisticated with emphasis on units and conversion within systems of units. In order to allow for the development of Number and Operations, there are relatively few expectations in Geometry and Data and Probability in Grade 4.

NUMBER AND OPERATIONS

Understand and use number notation and place value

N.ME.04.01 Read and write numbers to 1,000,000; relate them to the quantities they represent; compare and order.

N.ME.04.02 Compose and decompose numbers using place value to 1,000,000's, e.g., 25,068 is 2 ten thousands, 5 thousands, 0 hundreds, 6 tens, and 8 ones.

N.ME.04.03 Understand the magnitude of numbers up to 1,000,000; recognize the place values of numbers, and the relationship of each place value to the place to its right, e.g., 1,000 is 10 hundreds.

Use factors and multiples

N.ME.04.04 Find all factors of a whole number up to 50, and list factor pairs.

N.ME.04.05 List the first ten multiples of a given one-digit whole number; determine if a whole number is a multiple of a given one-digit whole number and if a one-digit number is a factor of a given whole number.

N.MR.04.06 Know that some numbers including 2, 3, 5, 7, and 11 have exactly two factors (1 and the number itself) and are called prime numbers.

N.MR.04.07 Solve problems about factors and multiples, e.g., since $100 = 4 \times 25$, and $200 = 2 \times 100$, then $200 = 2 \times 4 \times 25 = 8 \times 25$.

Add and subtract whole numbers

N.FL.04.08 Add and subtract whole numbers fluently.

Multiply and divide whole numbers

N.ME.04.09 Multiply two-digit numbers by 2, 3, 4, and 5, using the distributive property, e.g., $21 \times 3 = (1 + 20) \times 3 = (1 \times 3) + (20 \times 3) = 3 + 60 = 63$

N.FL.04.10 Multiply fluently any whole number by a one-digit number, and a three-digit number by a two-digit number; for a two-digit by one-digit multiplication, use distributive property to develop meaning for the algorithm.

N.FL.04.11 Divide numbers up to four digits by one-digit numbers and by 10.

N.FL.04.12 Find unknowns in equations such as $a \div 10 = 25$; $125 \div b = 25$.

N.MR.04.13 Use the relationship between multiplication and division to simplify computations and check results, e.g., $6840 \div 20 = (6840 \div 10) \div 2 = 684 \div 2 = 342$.

N.FL.04.14 Solve applied problems involving whole number multiplication and division.

Read, interpret and compare decimal fractions

N.ME.04.15 Read and interpret decimals up to two decimal places; relate to money and place value decomposition.

N.ME.04.16 Know that terminating decimals represents fractions whose denominators are 10, 10×10, $10 \times 10 \times 10$, etc., e.g., powers of 10.

N.ME.04.17 Locate tenths and hundredths on a number line.

N.ME.04.18 Read, write, interpret, and compare decimals up to two decimal places.

N.MR.04.19 Write tenths and hundredths in decimal and fraction forms, and know the decimal equivalents for halves and fourths.

Understand fractions

N.ME.04.20 Understand fractions as parts of a set of objects.

N.MR.04.21 Explain why equivalent fractions are equal, using models such as fraction strips or the number line, for fractions with denominators of 12 or less, or equal to 100.

N.MR.04.22 Locate and compare fractions on the number line, including improper fractions and mixed numbers with denominators of 12 or less.

N.MR.04.23 Understand the relationships among halves, fourths and eighths and among thirds, sixths and twelfths.

N.MR.04.24 Know that fractions of the form where $\frac{m}{n}$, is greater than n, are greater than 1 and are called improper fractions; locate improper fractions on the number line; express as mixed numbers.

N.MR.04.25 Write improper fractions as mixed numbers, and understand that a mixed number represents the number of "wholes" and the part of a whole remaining, e.g., $\frac{5}{4} = 1 + \frac{1}{4} = 1\frac{1}{4}$.

N.MR.04.26 Compare and order up to three fractions with denominators 2, 4, and 8, and 3, 6, and 12, including improper fractions and mixed numbers.

Add and subtract fractions

N.MR.04.27 Add and subtract fractions less than 1 with denominators 12 or less and including 100, in cases where the denominators are equal or when one denominator is a multiple ofthe other, e.g., 1/12 + 5/12 = 6/12; 2/25 + 7/50 = 11/50.

N.FL.04.28 Solve fraction problems involving sums and differences for fractions where one denominator is a multiple of the other (denominators 2 through 12, and 100).

N.MR.04.29 Solve for the unknown in equations such as: $\frac{1}{8} + x = \frac{5}{8}$ or $\frac{3}{4} - y = \frac{1}{2}$.

Multiply fractions by whole numbers

N.MR.04.30 Multiply fractions by whole numbers, using repeated addition and area or array models.

Add and subtract decimal fractions

N.MR.04.31 Use mathematical statements to represent problems that use addition and subtraction of decimals with up to two-digits; solve.

N.FL.04.32 Add and subtract decimals up to two decimal places.

Multiply and divide decimal fractions

N.FL.04.33 Multiply and divide decimals up to two decimal places by a one-digit whole number where the result is a terminating decimal, e.g., 0.42 ÷ 3 = 0.14, but not $5 \div 3 = 1.\overline{6}$

Estimate

N.FL.04.34 Estimate the answers to calculations involving addition, subtraction, or multiplication.

N.FL.04.35 Know when approximation is appropriate and use it to check the reasonableness of answers; be familiar with common place-value errors in calculations.

N.FL.04.36 Make appropriate estimations and calculations fluently with whole numbers using mental math strategies.

Problem-solving

N.MR.04.37 Solve applied problems using the four basic arithmetic operations for appropriate fractions, decimals, and whole numbers.

MEASUREMENT

Measure using common tools and appropriate units

M.UN.04.01 Measure using common tools and select appropriate units of measure.

M.PS.04.02 Give answers to a reasonable degree of precision in the context of a given problem and relative to the measurement tool used.

M.UN.04.03 Measure and compare integer temperatures in degrees, both Celsius and Fahrenheit.

M.TE.04.04 Measure surface area of cubes and rectangular prisms by covering and counting area of the faces.

Convert measurement units

M.TE.04.05 Carry out the following conversions from one unit of measure to a larger or smaller unit of measure: meters to centimeters, kilograms to grams, liters to milliliters, hours to minutes, minutes to seconds, years to months, weeks to days, feet to inches, ounces to pounds (using numbers that involve only simple calculations).

Use perimeter and area formulas

M.TE.04.06 Know and understand the formulas for perimeter and area of a square and a rectangle; calculate the perimeters and areas of these shapes and combinations of these shapes using the formulas.

M.TE.04.07 Find one dimension of a rectangle given the other dimension and its perimeter or area.

M.TE.04.08 Find the side of a square given its perimeter or area.

M.PS.04.09 Solve contextual problems about perimeter and area of squares and rectangles in compound shapes.

Understand right angles

M.TE.04.10 Identify right angles and compare angles to right angles.

Problem-solving

M.PS.04.11 Solve contextual problems about surface area.

GEOMETRY

Understand perpendicular, parallel, and intersecting lines

G.GS.04.01 Identify and draw perpendicular, parallel, and intersecting lines using a ruler and a tool or object with a square (90°) corner.

Identify basic geometric shapes and their components, and solve problems

G.GS.04.02 Identify basic geometric shapes including isosceles, equilateral and right triangles, and use their properties to solve problems.

G.SR.04.03 Identify and count the faces, edges, and vertices of basic three-dimensional geometric solids including cubes, rectangular prisms, and pyramids; describe the shape of their faces.

GEOMETRY

Recognize symmetry and transformations

G.TR.04.04 Recognize plane figures that have line symmetry.

G.TR.04.05 Recognize rigid motion transformations (flips, slides, turns) of a two-dimensional object.

DATA AND PROBOBILITY

Represent and solve problems for given data

D.RE.04.01 Construct tables and bar graphs from given data.

D.RE.04.02 Order a given set of data, find the median, and specify the range of values.

D.RE.04.03 Solve problems using data presented in tables and bar graphs, e.g., compare data represented in two bar graphs and read bar graphs showing two data sets.

FIFTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

5 MATH

v.4.04

NUMBER & OPERATIONS

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FIFTH GRADE

In the fifth grade, emphasis within the number area shifts to understanding of the addition and subtraction of fractions, with continued consolidation of multiplication and division concepts and skills with whole numbers. The idea of remainders in whole number division is addressed. Students learn the meaning of a fraction as the result of a division problem, and learn to work with decimals and percentages. In geometry and measurement, there is emphasis on the meaning and measurement of angles and on solving problems involving areas and angles. Work in number using exponents and factors begins to lead to algebraic ideas that will be more visible in grade six.

NUMBER AND OPERATIONS

Understand division of whole numbers

N.MR.05.01 Understand the meaning of division of whole numbers, with and without remainders; relate division to fractions and to repeated subtraction.

N.MR.05.02 Relate division of whole numbers with remainders to the form

$a = bq + r$, e.g., $34 \div 5 = 6$ r 4, so $5 \cdot 6 + 4 = 34$; note remainder (4) is less than divisor (6).

N.MR.05.03 Write mathematical statements involving division for given situations.

Multiply and divide whole numbers

N.FL.05.04 Multiply a multi-digit number by a two-digit number; recognize and be able to explain common computational errors such as not accounting for place value.

N.MR.05.05 Solve applied problems involving multiplication and division of whole numbers.

N.FL.05.06 Divide fluently up to a four-digit number by a two-digit number.

Find prime factorizations of whole numbers

N.MR.05.07 Find the prime factorization of numbers between 1 and 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number can be expressed as a product of primes.

Understand meaning of decimal fractions and percentages

N.ME.05.08 Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.

N.ME.05.09 Understand percentages as parts out of 100, use % notation, and express a part of a whole as a percentage.

Understand fractions as division statements; find equivalent fractions

N.ME.05.10 Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$ using simple fractions and pictures to represent.

N.ME.05.11 Given two fractions, express them as equivalent fractions with a common denominator, but not necessarily a <u>least</u> common denominator, e.g., $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{4} = \frac{6}{8}$; use denominators less than 12 or factors of 100.

Multiply and divide fractions

N.FL.05.12 Find the product of two unit fractions with small denominators using area model.

N.FL.05.13 Divide a fraction by a whole number and a whole number by a fraction using simple unit fractions.

Add and subtract fractions using common denominators

N.FL.05.14 Add and subtract fractions with unlike denominators of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 100, using the common denominator that is the product of the denominators of the 2 fractions, e.g.,

$$\frac{3}{8} + \frac{7}{10} = \frac{(3 \times 10) + (7 \times 8)}{80}$$

$$= \frac{30 + 56}{80}$$

$$= \frac{86}{80}$$

Multiply and divide by powers of ten

N.MR.05.15 Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, 1,000; and identify patterns.

N.FL.05.16 Divide numbers by 10's, 100's, 1,000's, using mental strategies.

N.MR.05.17 Multiply one-digit whole numbers by decimals up to two decimal places.

Solve applied problems with fractions

N.FL.05.18 Given an applied situation involving addition and subtraction of fractions, write mathematical statements describing the situation.

N.MR.05.19 Solve word problems that involve finding sums and differences of fractions with unlike denominators using knowledge of equivalent fractions.

N.FL.05.20 Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.

N.MR.05.21 Solve for the unknown in such equations as: $\frac{1}{4} + x = \frac{7}{12}$.

Express, interpret, and use ratios; find equivalences

N.MR.05.22 Express fractions and decimals as percentages and vice versa.

N.ME.05.23 Express ratios in several ways given applied situations, e.g., 3 cups to 5 people, 3 : 5, 3/5; recognize and find equivalent ratios.

MEASUREMENT

Know, and convert among, measurement units within a given system

M.UN.05.01 Recognize the equivalence of 1 liter, 1.000 ml and 1000 cm^3 and include conversions among liters, milliliters, and cubic centimeters.

M.UN.05.02 Know the units of measure of volume: cubic centimeter, cubic meter, cubic inches, cubic feet, cubic yards, and use their abbreviations (cm^3, m^3, in^3, ft^3, yd^3).

M.UN.05.03 Compare the relative sizes of one cubic inch to one cubic foot, and one cubic centimeter to one cubic meter.

M.UN.05.04 Convert measurements of length, weight, area, volume, and time within a given system using easily manipulated numbers.

Find areas of geometric shapes using formulas

M.PS.05.05 Represent relationships between areas of rectangles, triangles, and parallelograms using models.

M.TE.05.06 Understand and know how to use the area formula of a triangle:

$A = \frac{1}{2}bh$ (where b is length of the base and h is the height), and represent using models and manipulatives.

M.TE.05.07 Understand and know how to use the area formula for a parallelogram:

$A = bh$, and represent using models and manipulatives.

Understand the concept of volume

M.TE.05.08 Build solids with unit cubes and state their volumes.

M.TE.05.09 Use filling (unit cubes or liquid), and counting or measuring to find the volume of a cube and rectangular prism.

M.TE.05.10 Solve applied problems about the volumes of rectangular prisms using multiplication and division and using the appropriate units.

GEOMETRY

Know the meaning of angles, and solve problems

G.TR.05.01 Associate an angle with a certain amount of turning; know that angles are measured in degrees; understand that 90°, 180°, 270°, and 360° are associated, respectively, with 1/4, 1/2, and 3/4 and full turns.

G.GS.05.02 Measure angles with a protractor, and classify them as acute, right, obtuse, or straight.

G.GS.05.03 Identify and name angles on a straight line and vertical angles.

G.GS.05.04 Find unknown angles in problems involving angles on a straight line, angles surrounding a point and vertical angles.

G.GS.05.05 Know that angles on a straight line add up to 180° and angles surrounding a point add up to 360°; justify informally by "surrounding" a point with angles.

G.GS.05.06 Understand why the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is 360°, and use these properties to solve problems.

Solve problems about geometric shapes

G.GS.05.07 Find unknown angles using the properties of: triangles, including right, isosceles, and equilateral triangles; parallelograms, including rectangles and rhombuses; and trapezoids.

DATA AND PROBABILITY

Construct and interpret line graphs

D.RE.05.01 Read and interpret line graphs, and solve problems based on line graphs, e.g., distance-time graphs, and problems with two or three line graphs on same axes, comparing different data.

D.RE.05.02 Construct line graphs from tables of data; include axis labels and scale.

Find and interpret mean and mode for a given set of data

D.AN.05.03 Given a set of data, find and interpret the mean (using the concept of fair share) and mode.

D.AN.05.04 Solve multi-step problems involving means.

SIXTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

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v.4.04

NUMBER & OPERATIONS

ALGEBRA

MEASUREMENT

GEOMETRY

DATA & PROBABILITY

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SIXTH GRADE

Work with number is essentially completed by the end of sixth grade, where students' knowledge of whole numbers and fractions (ratios of whole numbers with non-zero denominators) should be introduced to integers and rational numbers. All of the number emphasis is intended to lay a foundation for the algebra expectations that are included in grade six. Students should use variables, write simple expressions and equations, and graph linear relationships. In geometry, students continue to expand their repertoire about shapes and their properties.

NUMBER AND OPERATIONS

Multiply and divide fractions

N.MR.06.01 Understand division of fractions as the inverse of multiplication, e.g., if $\frac{4}{5} \div \frac{2}{3} = \square$, then $\frac{2}{3} \times \square = \frac{4}{5}$, so $\square = \frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$.

N.FL.06.02 Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation.

N.MR.06.03 Solve for the unknown in equations such as: $\frac{1}{4} \div \square = 1\frac{3}{4} \div \square = \frac{1}{4}$ and $\frac{1}{2} = 1 \cdot \square$.

N.FL.06.04 Multiply and divide any two fractions, including mixed numbers, fluently.

Represent rational numbers as fractions or decimals

N.ME.06.05 Order rational numbers and place them on the number line.

N.ME.06.06 Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations.

N.ME.06.07 Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-\frac{8}{3}$ is -8 divided by 3.

Add and subtract integers and rational numbers

N.MR.06.08 Understand integer subtraction as the inverse of integer addition; add and subtract integers using integers from 10 to -10.

N.FL.06.09 Add, subtract, multiply, and divide integers between -10 and 10; use number line and strip models for addition and subtraction.

N.FL.06.10 Add, subtract, multiply and divide positive rational numbers fluently.

Find equivalent ratios

N.ME.06.11 Find equivalent ratios by scaling up or scaling down.

Solve decimal, percentage and rational number problems

N.FL.06.12 Calculate part of a number given the percentage and the number.

N.FL.06.13 Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers.

N.FL.06.14 For applied situations, estimate the answers to calculations involving operations with rational numbers.

N.FL.06.15 Solve applied problems that use the four operations with appropriate decimal numbers.

Use exponents

N.ME.06.16 Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.

Understand rational numbers and their location on the number line

N.ME.06.17 Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposite sides and at equal distance from 0 on a number line.

N.ME.06.18 Understand that rational numbers are quotients of integers (non-zero denominators), e.g., a rational number is either a fraction or a negative fraction.

N.ME.06.19 Understand that 0 is an integer that is neither negative nor positive.

N.ME.06.20 Know that the absolute value of a number is the value of the number, ignoring the sign, or is the distance of the number from 0.

ALGEBRA

Calculate rates

A.PA.06.01 Solve applied problems involving rates including speed, e.g., if a car is going 50 mph, how far will it go in $3\frac{1}{2}$ hours?

Understand the coordinate plane

A.RP.06.02 Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.

Use variables, write expressions and equations, and combine like terms

A.FO.06.03 Use letters, with units, to represent quantities in a variety of contexts, e.g., y lbs., k minutes, x cookies.

A.FO.06.04 Distinguish between an algebraic expression and an equation.

A.FO.06.05 Use standard conventions for writing algebraic expressions, e.g., $2x + 1$ means "two times x, plus 1" and $2(x + 1)$ means "two times the quantity $(x + 1)$."

A.FO.06.06 Represent information given in words using algebraic expressions and equations.

A.FO.06.07 Simplify expressions of the first degree by combining like terms, and evaluate using specific values.

Represent linear functions using tables, equations, and graphs

A.RP.06.08 Understand that relationships between quantities can be suggested by graphs and tables.

A.PA.06.09 Graph and write equations for linear functions of the form $y = mx$, and solve related problems, e.g., given n chairs, the "leg function" is $f(n) = 4n$; if you have 5 chairs, how many legs?; if you have 12 legs, how many chairs?

A.RP.06.10 Represent simple relationships between quantities, e.g., perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches; use verbal descriptions, formulas or equations, tables, and graphs.

Solve equations

A.FO.06.11 Relate simple linear equations with integer coefficients to particular contexts, and solve, e.g., $3x = 8$ or $x + 5 = 10$.

A.FO.06.12 Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.

A.FO.06.13 Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions.

A.FO.06.14 Solve equations of the form $ax + b = c$, e.g., $3x + 8 = 15$ by hand for positive integer coefficients less than 20, using calculators otherwise, and interpret the results.

MEASUREMENT

Convert within measurement systems

M.UN.06.01 Convert between basic units of measurement within a single measurement system, e.g., square inches to square feet.

Find volume and surface area

M.PS.06.02 Draw patterns (of faces) for a cube and rectangular prism that, when cut, will cover the solid exactly (nets).

M.TE.06.03 Compute the volume and surface area of cubes and rectangular prisms given the lengths of their sides using formulas.

GEOMETRY

Understand and apply basic properties

G.GS.06.01 Understand and apply basic properties of lines, angles, and triangles, including:

—triangle inequality

—relationships of vertical angles, complementary angles, supplementary angles

—congruence of corresponding and alternate interior angles when parallel lines are cut by a transversal, and that such congruencies imply parallel lines

—locate interior and exterior angles of any triangle, and use the property that an exterior angle of a triangle is equal to the sum of the remote (opposite) interior angles

—know that the sum of the exterior angles of a convex polygon is 360°.

Understand the concept of congruence and basic transformations

G.GS.06.02 Understand that for polygons, congruence means corresponding sides and angles have equal measures.

G.TR.06.03 Understand the basic rigid motions in the plane (reflections, rotations, translations), relate these to congruence, and apply them to solve problems.

G.TR.06.04 Understand and use simple compositions of basic rigid transformations, e.g., a translation followed by a reflection.

Construct geometric shapes

G.SR.06.05 Use paper folding to perform basic geometric constructions of perpendicular lines, midpoints of line segments and angle bisectors; justify informally.

DATA AND PROBABILITY

Understand the concept of probability and solve problems

D.PR.06.01 Express probabilities as fractions, decimals or percentages between 0 and 1; know that 0 probability means an event will not occur and that probability 1 means an event will occur.

D.PR.06.02 Compute probabilities of events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.

SEVENTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

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SEVENTH GRADE

The main focus in grade seven is the algebra concept of linear relationships including ideas about proportional relationships. Students should understand the relationship of equations to their graphs, as well as to tables and contextual situations for linear functions. In addition, work in algebra extends into simplifying and solving simple expressions and equations. The main concept from geometry in grade seven is similarity of polygons which also draws on ideas about proportion. Students apply their understanding of ratio in data-based situations.

NUMBER AND OPERATIONS

Understand derived quantities

N.ME.07.01 Understand derived quantities such as density, velocity, and weighted averages.

N.FL.07.02 Solve problems involving derived quantities.

Understand and solve problems involving rates, ratios, and proportions

N.FL.07.03 Calculate rates of change including speed.

N.MR.07.04 Convert ratio quantities between different systems of units such as feet per second to miles per hour.

N.FL.07.05 Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation a/b = c/d; know how to see patterns about proportional situations in tables.

Recognize irrational numbers

N.MR.07.06 Understand the concept of square root and cube root, and estimate using calculators.

Compute with rational numbers

N.FL.07.07 Solve problems involving operations with integers.

N.FL.07.08 Add, subtract, multiply and divide negative rational numbers.

N.FL.07.09 Estimate results of computations with rational numbers.

ALGEBRA

Understand and apply directly proportional relationships and relate to linear relationships

A.PA.07.01 Recognize when information given in a table, graph, or formula suggests a proportional or linear relationship.

A.RP.07.02 Represent directly proportional and linear relationships using verbal descriptions, tables, graphs, and formulas, and translate among these representations.

A.PA.07.03 Given a directly proportional or linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate $y = kx$ for specific x values, given k, e.g., weight vs. volume of water, base cost plus cost per unit.

A.PA.07.04 For directly proportional or linear situations, solve applied problems using graphs and equations, e.g., the heights and volume of a container with uniform cross-section, height of water in a tank being filled at a constant rate, degrees Celsius and degrees Fahrenheit, distance and time under constant speed.

A.PA.07.05 Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity.

Understand and represent linear functions

A.PA.07.06 Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that linear functions have slope that is a constant rate of change.

A.PA.07.07 Represent linear functions in the form $y = x + b$, $y = mx$, and $y = mx + b$, and graph, interpreting slope and y-intercept.

A.FO.07.08 Know that the solution to a linear equation corresponds to the point at which its graph crosses the x-axis.

Understand and solve problems about inversely proportional relationships

A.PA.07.09 Recognize inversely proportional relationships in contextual situations; know that quantities are inversely proportional if their product is constant, e.g., the length and width of a rectangle with fixed area, and that an inversely proportional relationship is of the form $y = k/x$ where k is some non-zero number.

A.RP.07.10 Know that the graph of $y = k/x$ is not a line; know its shape; and know that it crosses neither the x nor the y-axis.

Apply basic properties of real numbers in algebraic contexts

A.PA.07.11 Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition.

Combine algebraic expressions and solve equations

A.FO.07.12 Add, subtract, and multiply simple algebraic expressions of the first degree, e.g., $(92x + 8y) - 5x + y$, or $-2x(5x - 4)$, and justify using properties of real numbers.

A.FO.07.13 From applied situations, generate and solve linear equations of the form $ax + b = c$ and $ax + b = cx + d$, and interpret solutions.

GEOMETRY

Draw and construct geometric objects

G.SR.07.01 Use a ruler and other tools to draw squares, rectangles, triangles and parallelograms with specified dimensions.

G.SR.07.02 Use compass and straightedge to perform basic geometric constructions: the perpendicular bisector of a segment, an equilateral triangle, and the bisector of an angle; understand informal justifications.

Understand the concept of similar polygons, and solve related problems

G.TR.07.03 Understand that in similar polygons, corresponding angles are congruent and the ratios of corresponding sides are equal; understand the concepts of similar figures and scale factor.

G.TR.07.04 Solve problems about similar figures and scale drawings.

G.TR.07.05 Show that two triangles are similar using the criteria: corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity); use these criteria to solve problems and to justify arguments.

G.TR.07.06 Understand and use the fact that when two triangles are similar with scale factor of r, their areas are related by a factor of r^2.

Data and Probability

Represent data and interpret

D.RE.07.01 Represent and interpret data using circle graphs, stem and leaf plots, histograms, and box-and-whisker plots, and select appropriate representation to address specific questions.

D.AN.07.02 Create and interpret scatter plots and find line of best fit and use an estimated line of best fit to answer questions about the data.

Compute statistics about datasets

D.AN.07.03 Calculate and interpret relative frequencies and cumulative frequencies for given data sets.

D.AN.07.04 Find and interpret the median, quartiles, and interquartile range of a given set of data.

EIGHTH GRADE MATHEMATICS

GRADE LEVEL CONTENT EXPECTATIONS

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NUMBER & OPERATIONS

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EIGHTH GRADE

The main areas of emphasis for eighth grade are algebra, geometry, and data. In Algebra, students explore non-linear functions with a concentration on quadratic functions. Functions are studied through various representations: symbolic, graphical, with tables, and verbal statements. In Geometry, students consolidate their knowledge of two- and three-dimensional shapes and their properties, and apply this knowledge to perimeter, area, volume, and visualization problems. In Data and Probability, students take the basic knowledge gained to this point, and apply it to making decisions and performing basic analyses of data sets. Common probability misconceptions and bias in data presentation are addressed. Throughout these areas, students use their understanding of number and measurement, developed primarily prior to eighth grade, to solve more complex problems.

NUMBER AND OPERATIONS

Understand real number concepts

N.ME.08.01 Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube.

N.ME.08.02 Understand meanings for zero and negative integer exponents.

N.ME.08.03 Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g., $0.\overline{1}=\frac{1}{9}$; $0.\overline{3}=\frac{1}{3}$.

N.ME.08.04 Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g., $\sqrt{2}$, $\sqrt{3}$, π on the number line.

N.FL.08.05 Estimate and solve problems with square roots and cube roots using calculators.

N.FL.08.06 Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g., $\sqrt{130}$ is between 11 and 12.

Solve problems

N.MR.08.07 Understand percent increase and percent decrease in both sum and product form, e.g., 3% increase of a quantity x is x + .03x = 1.03x.

N.MR.08.08 Solve problems involving percent increases and decreases.

N.FL.08.09 Solve problems involving compounded interest or multiple discounts.

N.MR.08.10 Calculate weighted averages such as course grades, consumer price indices, and sports ratings.

N.MR.08.11 Solve problems involving ratio units such as miles per hour, dollars per pound, or persons per square mile.

Understand the concept of non-linear functions using basic examples

A.RP.08.01 Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions ($y = k/x$), cubics ($y = ax^3$) roots, ($y = \sqrt{x}$), and exponentials ($y = a^x$, $a > 0$), using tables, graphs, and equations.

A.PA.08.02 For basic functions, e.g., simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

A.PA.08.03 Recognize basic functions in problem context, e.g., area of a circle is πr^2, volume of a sphere is $\frac{4}{3}\pi r^3$, and represent them using tables, graphs, and formulas.

A.RP.08.04 Use the vertical line test to determine if a graph represents a function in one variable.

Understand and represent quadratic functions

A.RP.08.05 Relate quadratic functions in factored form and vertex form to their graphs and vice versa; in particular, note that solutions of a quadratic equation are the x-intercepts of the corresponding quadratic function.

A.RP.08.06 Graph factorable quadratic functions, finding where the graph intersects the x axis and the coordinates of the vertex; use words "parabola" and "roots"; include functions in vertex form and those with leading coefficient –1, e.g., $y = x^2 - 36$, $y = (x - 2)^2 - 9$; $y = -x^2$; $y = -(x - 3)^2$.

Recognize, represent, and apply common formulas

A.FO.08.07 Recognize and apply the common formulas:

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$; represent geometrically.

A.FO.08.08 Factor simple quadratic expressions with integer coefficients, e.g.,

$x^2 + 6x + 9$, $x^2 + 2x - 3$ and $x^2 - 4$; solve simple quadratic equations, e.g., $x^2 = 16$ or $x^2 = 5$ (by taking square roots); $x^2 - x - 6 = 0$, $x^2 - 2x = 15$ (by factoring); verify solutions by evaluation.

A.FO.08.09 Solve applied problems involving simple quadratic equations.

Understand solutions and solve equations, simultaneous equations, and linear inequalities

A.FO.08.10 Understand that to solve the equation $f(x) = g(x)$ means to find all values of x for which the equation is true, e.g., determine whether a given value, or values from a given set, is a solution of an equation (0 is a solution of $3x^2 + 2 = 4x + 2$, but 1 is not a solution).

A.FO.08.11 Solve simultaneous linear equations in two variables by graphing, by substitution, and by linear combination; estimate solutions using graphs; include examples with no solutions and infinitely many solutions.

A.FO.08.12 Solve linear inequalities in one and two variables, and graph the solution sets.

A.FO.08.13 Set up and solve applied problems involving simultaneous linear equations and linear inequalities.

GEOMETRY

Understand and use the Pythagorean Theorem

G.GS.08.01 Understand at least one proof of the Pythagorean Theorem; use the Pythagorean Theorem and its converse to solve applied problems including perimeter, area, and volume problems.

G.LO.08.02 Find the distance between two points on the coordinate plane using the distance formula; recognize that the distance formula is an application of the Pythagorean Theorem.

Solve problems about geometric figures

G.SR.08.03 Understand the definition of a circle; know and use the formulas for circumference and area of a circle to solve problems.

G.SR.08.04 Find area and perimeter of complex figures by sub-dividing them into basic shapes (quadrilaterals, triangles, circles).

G.SR.08.05 Solve applied problems involving areas of triangles, quadrilaterals, and circles.

Understand concepts of volume and surface area, and apply formulas

G.SR.08.06 Know the volume formulas for generalized cylinders ((area of base) × height), generalized cones and pyramids ($\frac{1}{3}$(area of base) × height) and spheres ($\frac{4}{3}\pi$ (radius)^3) and apply them to solve problems.

G.SR.08.07 Understand the concept of surface area, and find the surface area of prisms, cones, spheres, pyramids, and cylinders.

Visualize solids

G.SR.08.08 Sketch a variety of two-dimensional representations of three-dimensional solids including orthogonal views (top, front, and side), picture views (projective or isometric), and nets, use such two-dimensional representations to help solve problems.

Understand and apply concepts of transformation and symmetry

G.TR.08.09 Understand the definition of a dilation from a point in the plane, and relate it to the definition of similar polygons.

G.TR.08.10 Understand and use reflective and rotational symmetries of two-dimensional shapes, and relate them to transformations to solve problems.

DATA AND PROBABILITY

Draw, explain, and justify conclusions based on data

D.AN.08.01 Determine which measure of central tendency (mean, median, mode) best represents a data set, e.g., salaries, home prices for answering certain questions; justify the choice made.

D.AN.08.02 Recognize practices of collecting and displaying data that may bias the presentation or analysis.

Understand probability concepts for simple and compound events

D.PR.08.03 Compute relative frequencies from a table of experimental results for a repeated event, and be able to answer questions about the result, using relationship of probability to relative frequency.

D.PR.08.04 Apply the Basic Counting Principle to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.

D.PR.08.05 Understand the relationship of probability to relative frequency.

D.PR.08.06 Understand the difference between independent and dependent events, and recognize common misconceptions involving probability, e.g., Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the fourth roll as she was on any previous roll.

D.AN.08.07 Compute relative frequencies from a table of experimental results for a repeated event; understand the relationship of experimental probability to relative frequency; answer questions regarding the results.